Name:_____ Date:_____

Pre-Calculus 11: HW 1.3 Geometric Sequences Solution:

1. Which of the following sequences is geometric. Indicate YES or NO: If YES, indicate the common ratio "*r*":

a) $2,\ 4,\ 6,\ 8,\ 10.....$

This is not geometric, it's arithmetic! Each term is adding by 2. To be geometric, it needs to multiply by a common ratio

b) $8,\ 12,\ 18,\ 27,\ 40.5.....$

This sequence is geometric.

$\frac{12}{8}=1.5,\ \frac{18}{12}=1.5,\ \frac{27}{18}=1.5,\ \frac{40.5}{27}=1.5$

The common ratio is 1.5

c) $0.25,\ 0.50,\ 1.0,\ 2.0,\ 4.0$

This sequence is geometric.

$\frac{0.5}{0.25}=2,\ \frac{1}{0.5}=2,\ \frac{2}{1}=2,\ \frac{4}{2}=2$

The common ratio is 2

d) $\frac{2}{3},\ \frac{-1}{3},\ \frac{1}{6},\ \frac{-1}{12},\ \frac{1}{24}$

This sequence is geometric.

$\frac{-1/3}{2/3}=-0.5,\ \frac{1/6}{-1/3}=-0.5,\ \frac{-1/12}{1/6}=-0.5,\ \frac{1/24}{-1/12}=-0.5$

The common ratio is -0.5

e) $-4,-8,\ 16,\ 32,-64.....$

This sequence is not geometric.

$\frac{-8}{-4}=2,\ \frac{16}{-8}=-2,\ \frac{32}{16}=2,\ \frac{-64}{32}=-2$

The common ratio is inconsistent

f) $3,\ 9,\ 27,\ 81,\ 243,...$

This sequence is geometric.

$\frac{9}{3}=3,\ \frac{27}{9}=3,\ \frac{81}{27}=3,\ \frac{243}{81}=3$

The common ratio is 3

g) $0.75,\ -0.75,\ 0.75,\ -0.75,\ 0.75$

This sequence is geometric.

$\frac{-0.75}{0.75}=-1,\ \frac{0.75}{-0.75}=-1,\ \frac{-0.75}{0.75}=-1,\ \frac{0.75}{-0.75}=-1$

The common ratio is -1

h) $\frac{27}{32},\ \frac{9}{16},\ \frac{3}{8},\ \frac{1}{4},\ \frac{1}{6}$

This sequence is geometric.

$\frac{9/16}{27/32}=\frac{2}{3},\ \frac{3/8}{9/16}=\frac{2}{3},\ \frac{1/4}{3/8}=\frac{2}{3},\ \frac{1/6}{1/4}=\frac{2}{3}$

The common ratio is 2/3

2. If the following is a geometric sequence, indicate the number of terms:

a) $6,\ 12,\ 24,\ .....,\ 3072$

$a=6, r=2, t_n=3072, n=?$

$t_n=a\times(r)^{n-1}$

$3072=6\times(2)^{n-1}$

$512=2^{n-1}$

$2^9=2^{n-1}\qquad 10=n$

$9=n-1$

b) $24,\ 12,\ 6,\ .....,\ \frac{3}{512}$

$a=24, r=0.5, t_n=\frac{3}{512}, n=?$

$t_n=a\times(r)^{n-1}$

$\frac{3}{512}=24\times(0.5)^{n-1}$

$\frac{1}{2048}=2^{n-1}$

$2^{-11}=2^{n-1}\qquad -10=n$

$-11=n-1$

c) $\sqrt{3},\ -3,\ 3\sqrt{3},\ .....,\ 243\sqrt{3},$

$a=\sqrt{3}, r=-\sqrt{3}, t_n=243\sqrt{3}, n=?$

$t_n=a\times(r)^{n-1}$

$243\sqrt{3}=\sqrt{3}\times(-\sqrt{3})^{n-1}$

$243=(-\sqrt{3})^{n-1}$

$\sqrt{3}^{10}=(-\sqrt{3})^{n-1}\qquad 11=n$

$10=n-1$

d) $\frac{1}{8},\ -0.25,\ 0.5,\ .....,\ -1024$

$a=\frac{1}{8}, r=-2, t_n=-1024, n=?$

$t_n=a\times(r)^{n-1}$

$-1024=\frac{1}{8}\times(-2)^{n-1}$

$-8192=(-2)^{n-1}$

$-2^{13}=(-2)^{n-1}\qquad 14=n$

$13=n-1$

e) $396,\ -132,\ 44,\ \ldots\ldots,\ \dfrac{44}{729}$

$a=396, r=\dfrac{1}{3}, t_n=\dfrac{44}{729}, n=?$

$t_n = a\times(r)^{n-1}$

$\dfrac{44}{729} = 396\times\left(\dfrac{1}{3}\right)^{n-1}$

$\dfrac{1}{729} = 9\left(\dfrac{1}{3}\right)^{n-1}$

$\dfrac{1}{6561} = \left(\dfrac{1}{3}\right)^{n-1} \qquad 8 = n-1$

$\left(\dfrac{1}{3}\right)^{8} = \left(\dfrac{1}{3}\right)^{n-1} \qquad 9 = n$

f) $2048,\ 512,\ 128,\ \ldots\ldots,\ \dfrac{1}{2048}$

$a=2048, r=\dfrac{1}{2}, t_n=\dfrac{1}{2048}, n=?$

$t_n = a\times(r)^{n-1}$

$\dfrac{1}{2048} = 2048\times\left(\dfrac{1}{2}\right)^{n-1}$

$\dfrac{1}{4194304} = \left(\dfrac{1}{2}\right)^{n-1}$

$\left(\dfrac{1}{2}\right)^{22} = \left(\dfrac{1}{2}\right)^{n-1} \qquad 23 = n$

$22 = n-1$

3. Given the information of a geometric sequence, find the indicated unknown value. Show your work algebraically:

a) $a=-3,\ r=5,\ t_4=$

$t_n = a(r)^{n-1}$

$t_4 = -3(5)^{4-1}$

$t_4 = -3(5)^3$

$t_4 = -375$

b) $a=16,\ r=-0.5,\ t_7=$

$t_n = a(r)^{n-1}$

$t_7 = 16(-0.5)^{7-1}$

$t_7 = 16(-0.5)^6$

$t_7 = 0.25$

c) $a=\dfrac{1}{6},\ t_6=40.5,\ r=$

$t_n = a(r)^{n-1}$

$40.5 = \dfrac{1}{6}(r)^{6-1}$

$243 = (r)^5$

$\sqrt[5]{243} = r$

$3 = r$

d) $a=24,\ t_5=\dfrac{1}{6},\ r=$

$t_n = a(r)^{n-1}$

$\dfrac{1}{6} = 24(r)^{5-1}$

$144 = (r)^4$

$\sqrt[4]{144} = r$

$3.4641\ldots = r$

e) $t_3=36,\ t_4=54,\ r=?$

$t_4 = t_3\times r$

$54 = 36\times r$

$\dfrac{54}{36} = r$

$\dfrac{3}{2} = r$

f) $t_5=18, t_4=9, a=?$

$t_5 = a\times r^{n-1}$

$t_5 = t_4\times r$

$18 = 9\times r$

$2 = r$

$18 = a\times(2)^4$

$\dfrac{18}{16} = a$

$\dfrac{9}{8} = a$

g) $t_3=12,\ \ t_5=48,\ t_4=?$

$t_5 = t_3\times r^2$

$48 = 12\times r^2$

$4 = r^2$

$\pm 2 = r$

$t_4 = t_3\times r$

$t_4 = 12\times(\pm 2)$

$t_4 = \pm 24$

h) $t_6=432, t_4=48, a=?$

$t_6 = t_4\times r^2$

$432 = 48\times r^2$

$9 = r^2$

$\pm 3 = r$

$t_4 = a\times r^3$

$48 = a\times(\pm 3)^3$

$48 = a\times(\pm 27)$

$\pm\dfrac{48}{27} = \pm\dfrac{16}{9} = a$

4. What is the main difference between an arithmetic sequence versus a geometric sequence?

An arithmetic sequence will ***add*** the same difference with each successive term. An geometric sequence will ***multiply*** by the same ratio with each successive term.

5. Is the following a geometric sequence? Explain why or why not:
   i) $4, -4, 4, -4, 4, -4, ...$ ii) $7, 7, 7, 7, 7, 7, 7.....$

Both terms are geometric sequences because they have a common ratio. The first sequence has a common ratio of -1. The second geometric sequence has a common ratio of 1.

6. Suppose you had a generous friend that gave you money every day. On the first day he would give you a dollar and then each successive the amount increase by 20%. How much money will he give you on the 10th day?

$a = 1, r = 1.20, n = 10$

$t_{10} = 1 \times (1.20)^{10-1}$
$t_{10} = 1.20^{9}$
$t_{10} = \$5.16$

7. Suppose the population of a colony of cockroaches double every three days. If they begin with 2, how many will there be in 30 days?

$a = 2, \quad r = 2 \ (double), \quad n = \frac{30}{3} = 10$

$t_{10} = 2 \times (2)^{10-1}$
$t_{10} = 2 \times (2)^{9}$
$t_{10} = 1024$

8. If you don't pay your Visa bill, the bank will charge you 20% interest each month. The interest is compounded, meaning that they will charge you interest on previous interest. If your bill is \$100 and left unpaid, how much will you owe after 12 months?
   The first term is \$100, the common ratio is 1.20 because 20% is added onto what you already owe. The number of terms is 13 because it's after 12 months [1st month has no interest]
   $t_{13} = 100(1.2)^{13-1}$
   $t_{13} = 100(1.2)^{12}$
   $t_{13} = 891.61$
   If you don't pay your visa bill at 20% per month, you will owe \$891 after 12 months because of interest

9. The S&P 500, or the Standard & Poor's 500, is a stock market index based on the stock price of 500 large companies in the US. Many people use it as an indicator of the US stock market and it's economy. In January 1985, the price of the index fund was \$179. In January 1995, the price of the index fund was \$470. If the growth was modelled by a geometric sequence, determine the value of the index fund in 2005 and 2015.

   Since the increments are in 10 years, let index price in 1985 be t_1, 1995 be t_2, 2005 be t_3, and 2015 be t_4

   $a = 179, t_2 = 470$
   $r = \frac{t_2}{t_1} = \frac{470}{179}$
   $r = 2.62569832.....$

   $t_3 = 470 \times 2.62569832...$
   $t_3 = \$1234.07$
   $t_4 = 1234.07 \times 2.62569832...$
   $t_4 = \$3240.32$

10. What value of "x" in $x,\ 2x+2,\ 3x+3$ will form a geometric sequence?

If the three terms is a geometric sequence then:

$$\frac{2x+2}{x}=\frac{3x+3}{2x+2}$$

$$\frac{2(x+1)}{x}=\frac{3(x+1)}{2(x+1)}$$

$$4(x+1)=3x$$

$$4x+4=3x$$

$$x=-4$$

11. Determine the value of "*x*" which makes $3, 3^x, 3^{x-5}$ a geometric sequence?

If the three terms is a geometric sequence then:

$$\frac{3^x}{3}=\frac{3^{x-5}}{3^x}$$

$$3^{x-1}=3^{x-5-x}$$

$$x-1=-5$$

$$x=-4$$

12. If $t_5=3x+2$ and $t_7=7x-22$ with a common ratio of $r=-3$, determine t_6 *and* t_8.

$$t_7=t_5\times r\times r$$

$$t_7=t_5\times r^2$$

$$7x-22=(3x+2)(-3)^2$$

$$7x-22=(3x+2)9$$

$$7x-22=27x+18$$

$$-40=20x$$

$$-2=x$$

$$t_5=3x+2=3(-2)+2$$

$$t_5=-4$$

$$t_6=-4\times(-3)=12$$

$$t_7=7x-22=7(-2)-22$$

$$t_7=-36$$

$$t_8=-36\times(-3)=108$$

13. Determine t_2 of a geometric sequence if $t_4+t_5=-3$ and $t_3+t_4=-6$

$$t_4=a\times r^3$$

$$t_5=a\times r^4$$

$$t_3=a\times r^2$$

$$ar^3+ar^4=-3$$

$$ar^3(1+r)=-3$$

$$a\times r^2+ar^3=-6$$

$$ar^2(1+r)=-6$$

$$\frac{ar^3(1+r)}{ar^2(1+r)}=\frac{-3}{-6}$$

$$r=0.5$$

$$ar^2+ar^3=-6$$

$$a(0.5)^2+a(0.5)^3=-6$$

$$a[0.25+0.125]=-6$$

$$a(0.375)=-6$$

$$a=-16$$

$$t_2=a\times r$$

$$t_2=0.5\times(-16)$$

$$t_2=-8$$